Some considerations on evolution and lightcone effects for BLSS applications

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Some notes on the issue of how to treat evolving number density and bias in BLSS applications. Here, results are written in terms of the perturbative bias expansion (this note originates from another note which was eventually intended to become a paper...). Everything can be simply generalized to a bias function $F(\delta_m)$, for example, although the result is not guaranteed to be a correct description of galaxies. The conclusions are: 1. There is no need to distinguish between radial selection function and mean number density, as the selection can be incorporated into the bias parameters. 2. It is preferable to use the mean galaxy density $\bar{n}_g(z)$ inferred from the data in fine-grained z bins. If a constant $\bar{n}_g$ is assumed for example, this leads to large line-of-sight perturbations in δ_g, which are in general inconsistent with the prior on $P(k)$. Even when using a $\bar{n}_g(z)$ measured from the data, it is certainly a good idea to allow for an overall $\bar{n}_g$ to be a free parameter in order to absorb any unmodeled $k = 0$ mode.

When describing observed galaxy samples, two effects need to be considered:

1. Sample selection: in practice, tracers are chosen based on some observable property (luminosity, color, ...). Of course, there is significant scatter in the relation of these quantities to say, halo mass. How does this scatter enter into the bias expansion ? It is trivially incorporated, as described in Sec. I (see also Sec. 9.2 in [1]).

2. Light-cone effects: large surveys cover a volume with significant extent along the line-of-sight direction. Thus, one cannot assume that all tracers are measured on a fixed time slice, and has to take into account that their number density and bias parameters evolve.

I. SAMPLE SELECTION (SEE ALSO SEC. 9.2 IN [1])

Let α denote a physical observable (say, luminosity) that is used to select tracers. Given a selection function $S(\alpha)$, the observed galaxy density at position $\boldsymbol{x}, \tau$ is (we neglect projection effects including RSD throughout)

$$n_g^{\rm obs}(\boldsymbol{x}, \tau) = \int_\alpha S(\alpha) n_g(\boldsymbol{x}, \tau; \alpha) = \int_\alpha S(\alpha) \bar{n}_g(\tau; \alpha) \left[1 + \delta_g(\boldsymbol{x}, \tau; \alpha)\right] . \tag{1}$$

Let us apply the bias expansion to the "conditional" overdensity $\delta_g(\boldsymbol{x}, \alpha)$:

$$\delta_g(\boldsymbol{x}, \tau; \alpha) = \sum_O b_O(\alpha) O(\boldsymbol{x}, \tau) , \tag{2}$$

where O are operators such as δ_m, $\delta_m^2 - \langle\delta_m^2\rangle$, and so on. Importantly, we assume throughout that $\langle\delta_g\rangle_{\rm survey} = 0$, which implies

$$\langle O\rangle_{\rm survey} = 0 . \tag{3}$$

Note that when adopting a non-perturbative bias model, such as $\delta_g = F(\delta_m)$, then this correspondingly requires $\langle F(\delta_m)\rangle_{\rm survey} = 0$.

Eq. (2) also takes into account that the distribution of α at each point $(\boldsymbol{x}, \tau)$ can depend on $\{O\}$ as well (more red galaxies in dense regions for example). Thus,

$$n_g^{\rm obs}(\boldsymbol{x}, \tau) = \bar{n}_g(\tau) \left[1 + \sum_O b_O^S \, O(\boldsymbol{x}, \tau)\right] , \tag{4}$$

with

$$b_O^S = \int_\alpha S(\alpha) b_O(\alpha) . \tag{5}$$

One further expects that the relation between observables and the physical properties of the halos that determine the bias parameters is stochastic. This is similarly absorbed into the stochastic fields which need to be included for a consistent bias expansion anyway. Thus, in the context of the general perturbative bias expansion, we do not have to worry about splitting the sample into individual physical populations. Moreover, there is no need to attempt to distinguish a "true" mean galaxy density from a selection function, as the effects of the selction are incorporated in the (z-dependent) bias parameters.

II. LIGHTCONE EFFECTS

Let us focus on the issue that, in a galaxy survey covering a certain redshift interval Δz, the population of galaxies evolves, commonly referred to as "lightcone effects". For this, we neglect all other projection effects, including RSD. We also assume a small survey area on the sky (or at least small pair separation angle) so that we can work in the flat-sky limit. The goal is to treat the measured galaxy density field on one effective time slice $\tau = \bar{\tau}$.

Following our approximations, the spacetime location of a galaxy detected at observed sky position $\hat{\boldsymbol{n}}$ and redshift z is

$$(\boldsymbol{x}, \tau) = (\boldsymbol{x}_o + \hat{\boldsymbol{n}}\chi(z), \tau_o - \chi(z)) , \tag{6}$$

where a subscript o denotes spacetime coordinates of the observer. We now recenter the spatial comoving coordinates to a convenient origin given by

$$\boldsymbol{o} = \bar{\boldsymbol{n}}\, \bar{\chi} , \quad \bar{\chi} \equiv \chi(\bar{z}) , \tag{7}$$

where $\bar{z}$ is a reference redshift that we leave arbitrary for now (reasonable choices will be defined below) and $\bar{\boldsymbol{n}}$ is a suitable center point of the survey footprint on the sky. Thus, in the following we let $\boldsymbol{x} \mapsto \boldsymbol{x} - \boldsymbol{o}$. In particular, the line-of-sight coordinate becomes

$$x_\parallel \equiv \bar{\boldsymbol{n}} \cdot \boldsymbol{x} = \chi(z) - \chi(\bar{z}) = \bar{\tau} - \tau , \tag{8}$$

where $\bar{\tau} = \tau_o - \chi(\bar{z})$ is the conformal time on the lightcone corresponding to the origin of the coordinate system. Note that the flat-sky approximation implies that $\hat{\boldsymbol{n}}$ is treated as constant over the entire survey.

The observed galaxy density (neglecting all projection effects, as mentioned above) is

$$n_g(\boldsymbol{x}, \tau) = \bar{n}_g(\tau) \left[1 + \delta_g(\boldsymbol{x}, \tau)\right] , \tag{9}$$

where un the general bias expansion, we write the galaxy density perturbation at a given spacetime location $(\boldsymbol{x}, \tau)$ as

$$\delta_g(\boldsymbol{x}, \tau) = \sum_O \left[b_O(\tau) + \epsilon_O(\boldsymbol{q}[\boldsymbol{x}, \tau], \tau)\right] O\left(\boldsymbol{q}[\boldsymbol{x}, \tau], \tau\right) , \tag{10}$$

where $\boldsymbol{q}[\boldsymbol{x}, \tau]$ denotes the Lagrangian position of the fluid trajectory passing through $(\boldsymbol{x}, \tau)$ (we will revert to Eulerian expressions in the end; for now Lagrangian coordinates make the derivation more clear). Using the coordinates defined above, we now have two options to simplify this description if we do not want to fully include lightcone effets:

1. Define

$$\delta_g^{\rm obs}(\hat{\boldsymbol{n}}, z) = \frac{n_g(\hat{\boldsymbol{n}}\chi(z) - \boldsymbol{o}, \bar{\tau} - x_\parallel)}{\bar{n}_g(\bar{\tau} - x_\parallel)} - 1 , \tag{11}$$

i.e. with respect to the mean density estimated for that redshift (e.g. by considering the total number of galaxies in fine bins in z).

2. Define

$$\delta_g^{\rm obs}(\hat{\boldsymbol{n}}, z) = \frac{n_g(\hat{\boldsymbol{n}}\chi(z) - \boldsymbol{o}, \bar{\tau} - x_\parallel)}{\bar{n}_g(\bar{\tau})} - 1 , \tag{12}$$

using the mean density at the pivot redshift $\bar{z}$. It could also be t galaxy density averaged over the entire survey volume. The key point is that a constant, rather than z-dependent value is used.

We consider both cases in turn.

1. In this case, we are normalizing the density using the "correct" mean density at each redshift (i.e., the best estimate within the survey volume). Hence, the *actual* galaxy density at the point $\boldsymbol{x}$ on the lightcone is in this case

$$\delta_g^{\rm obs}(\boldsymbol{x}) \overset{\rm actual}{=} \sum_O \left[b_O(\tau) + \epsilon_O(\boldsymbol{q}[\boldsymbol{x}, \tau], \tau)\right] O\left(\boldsymbol{q}[\boldsymbol{x}, \tau], \tau\right) \Big|_{\tau = \bar{\tau} - x_\parallel} . \tag{13}$$

On the other hand, when *ignoring* light-cone effects, we assume

$$\delta_g^{\rm obs}(\boldsymbol{x}) \overset{\rm ignoring\ l.c.}{=} \sum_O \left[b_O(\bar{\tau}) + \epsilon_O(\boldsymbol{q}[\boldsymbol{x}, \bar{\tau}], \bar{\tau})\right] O\left(\boldsymbol{q}[\boldsymbol{x}, \bar{\tau}], \bar{\tau}\right) . \tag{14}$$

The difference is that, first, we evaluate $\delta_g^{\rm lc}$ at time $\bar\tau$ instead of $\tau = \bar\tau - x_\parallel$. Second, we do not assign the galaxy the position $\boldsymbol{x}_{\rm fl} = \boldsymbol{x}_{\rm fl}(\boldsymbol{q}, \bar\tau)$ that the fluid trajectory actually passes through at time $\bar\tau$ (as we cannot simply reconstruct the fluid trajectories), but assign it $\boldsymbol{x}$, which differs from the former by

$$\boldsymbol{x} - \boldsymbol{x}_{\rm fl}(\boldsymbol{q}, \bar\tau) = \boldsymbol{s}(\boldsymbol{q}, \tau) - \boldsymbol{s}(\boldsymbol{q}, \bar\tau)\,, \tag{15}$$

where $\boldsymbol{s}$ is the Lagrangian displacement. One can then show that the leading corrections to Eq. (14) from the evolution of bias parameters as well as the displacement is of order $(\mathcal{H}x_\parallel)^2 \leq (\Delta z)^2 \times \delta_g$, where Δz is the full width of the redshift bin. When choosing $\Delta z \ll 1$, these are quite small, and the approximation Eq. (14) is sufficient for current surveys (e.g. CMASS).

2. The second option differs from the first by a factor $\bar n_g(\bar\tau - x_\parallel)/\bar n_g(\bar\tau)$. Instead of Eq. (13), we thus have

$$\begin{aligned} 1 + \delta_g^{\rm obs}(\boldsymbol{x})\Big|_{(2)} &\stackrel{\rm actual}{=} \frac{\bar n_g(\bar\tau - x_\parallel)}{\bar n_g(\bar\tau)} \left[1 + \sum_O \left[b_O(\tau) + \epsilon_O(\boldsymbol{q}[\boldsymbol{x},\tau],\tau)\right] O\left(\boldsymbol{q}[\boldsymbol{x},\tau],\tau\right)\Big|_{\tau=\bar\tau - x_\parallel}\right] \\ &= \left[\sum_{n=1}^\infty \bar n_g^{-1} \frac{d^n \bar n_g}{d(\ln a)^n}\Big|_{\bar\tau} (-\mathcal{H}x_\parallel)^n\right] \left[1 + \sum_O \left[b_O(\tau) + \epsilon_O(\boldsymbol{q}[\boldsymbol{x},\tau],\tau)\right] O\left(\boldsymbol{q}[\boldsymbol{x},\tau],\tau\right)\Big|_{\tau=\bar\tau - x_\parallel}\right]\,. \end{aligned} \tag{16}$$

The key difference to Eq. (13) is that now there is a modulation of $\delta_g^{\rm obs}$ that is *zeroth order* in perturbations. That is, the z-dependence of the mean (observed) galaxy density leads to a large-scale contribution to $\delta_g^{\rm obs}$. This needs to be carefully marginalized over (e.g., by not imposing a prior on large-scale line-of-sight modes). On the other hand, in option 1. above, the evolution effects only multiply perturbations, which greatly reduces their effect.

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